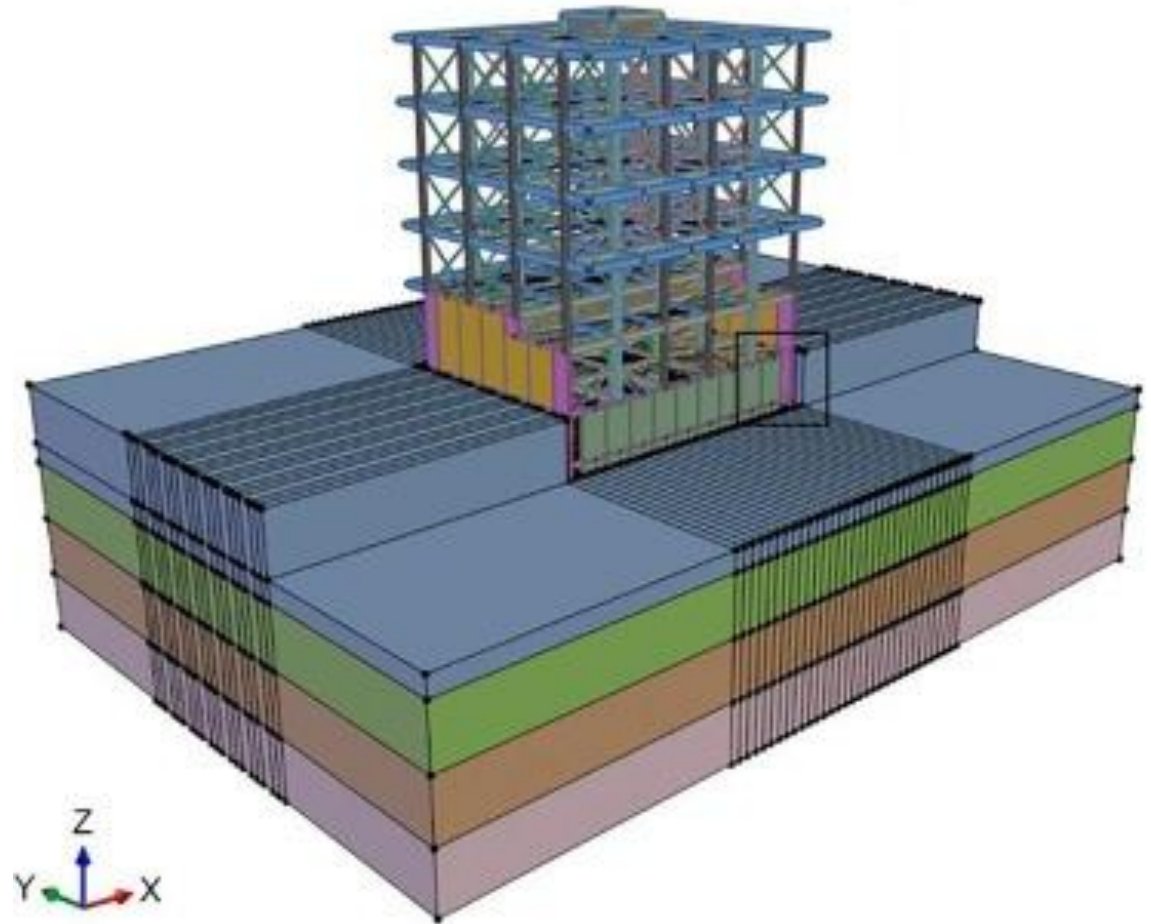
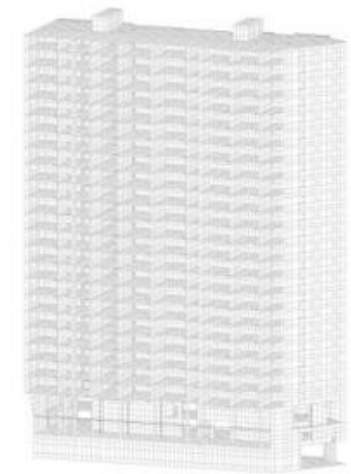


Topic 2. Development of 3D Soil Base Models



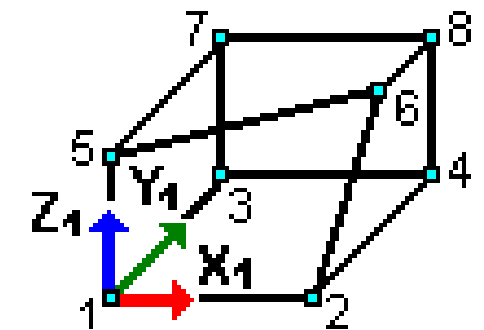
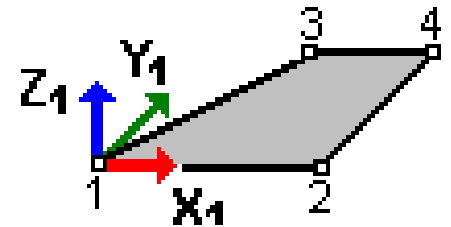
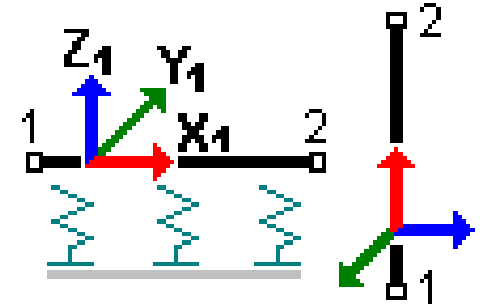
Finite elements for solving the volumetric problem of the theory of elasticity

Computational analysis of any design begins with an attempt to establish what is essential in this case, and what can be neglected. This kind of simplification of the problem is always carried out, since the calculation, taking into account all the properties of the real design, is possible only with a certain degree of approximation. The structural scheme of the structure is a simplified scheme of a real structure, reflecting only those parameters of the structure and the loads acting on it that are related to the solution of this problem. When calculating building structures and their foundations and foundations, for example, the requirements for design schemes are given in regulatory documents, which indicate that design schemes must reflect the actual operating conditions of the building or structure and correspond to the considered design situation, taking into account their design features.



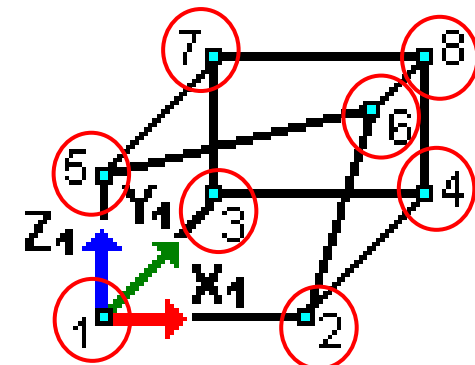
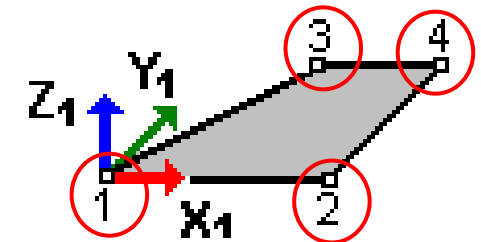
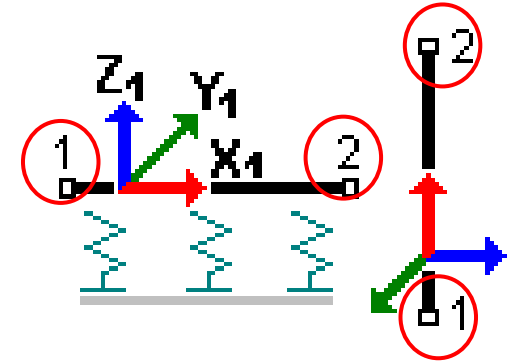
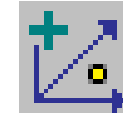
Some methods of constructing computational circuits are widespread and are of a general nature (idealization of a material in the form of a continuous medium; assumption of homogeneity of the material; reduction of the geometric shape of a body to such standard circuits as rods, plates or shells; schematization of external forces, etc.). Other methods of schematization are quite specific and are associated with each problem considered. However, in all cases, the choice of a design scheme is the most important element of analysis, one of the most characteristic features of engineering art and characterizes the level of professional skill of an engineer.

If the method of displacements is used to analyze the design scheme, then when idealizing the structure, the design scheme is presented as a set of bodies of the standard type (rods, plates, volumetric elements, etc.), which are called finite element elements (CE) and are connected to the nodes of the design scheme. The larger the finite element library (FSB) of a modern computing complex, the greater the capabilities of the engineer in modeling the structure.



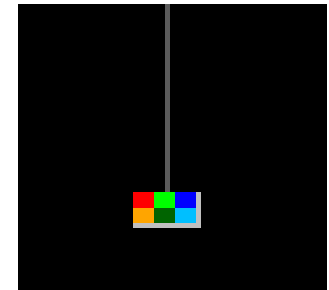
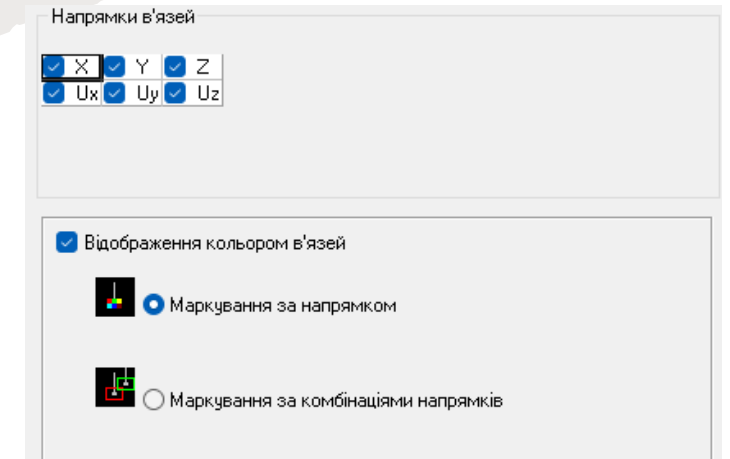
In the calculation scheme of the displacement method, the node is presented in the form of an absolutely rigid body of vanishingly small size. The position of the node in space during deformations of the system is determined by the coordinates of the center and the directions of the three axes rigidly connected to the node. In other words, a node is represented as an object that has six degrees of freedom for most problems: three linear displacements, defined as the difference in coordinates in the deformed and non-deformed states, and three rotation angles. In some cases, additional degrees of freedom are attributed to the node of the computing circuit, which are no longer identified with the components of linear or angular displacements of the node in the form of an infinitesimal rigid body at a point coinciding with the center of the node. These can be torsions of the deformed surface of the plate or components of transverse shear and compression deformations in layered lumpy heterogeneous inclined shells.

Unknown to the method of displacement are the values of the degrees of freedom of the nodes of the design scheme. Since the elements of the system are considered as attached only to its nodes, the problem of the continuum is reduced to a discrete one in the form of a system of equilibrium equations according to the degrees of freedom of the nodes (their directions).



Due to the peculiarities of the design solution, restrictions (knitting) may be imposed on some nodal movements. A system is called monomuscular if it prohibits one of the components of nodal movement (for example, when moving along the axis of OX). If, by virtue of the elm superimposed on the system, some linear combination of the constituent nodal displacements is zero, then we are talking about a field tie. The monoligament and polyovius superimposed on the node of displacement of the components of the same node are external.

The danger in creating a structural scheme lies in the omission of the elm - the absence of a ban on movement, which leads to the appearance of geometric variability.



Conditions for the adjoining of the same type of elements

Умови примикання стержнів

Звільнення лінійних в'язей

		Вузол 1		Вузол 2
		Жорсткість		Жорсткість
		T/м		T/м
X1	☐	0		☐ 0
Y1	☐	0		☐ 0
Z1	☐	0		☐ 0

Звільнення кутових в'язей

		Вузол 1		Вузол 2
		Жорсткість		Жорсткість
		T*м/град		T*м/град
UX1	☐	0		☐ 0
UY1	☐	0		☐ 0
UZ1	☐	0		☐ 0
D	☐			☐

☐ Позначити усе

☐ Позначити усе

Розташування шарніру

Вузол 1

☒ В околі переходу до гнучкої частини, в місцевій системі координат

☐ Біля вузла, в місцевій системі координат

☐ Біля вузла, в загальній системі координат

Вузол 2

☒ В околі переходу до гнучкої частини, в місцевій системі координат

☐ Біля вузла, в місцевій системі координат

☐ Біля вузла, в загальній системі координат

(spatial rods)

Умови примикання пластин

Звільнення в'язей

		Жорсткість
		T/м
X1	☐	0
Y1	☐	0
Z1	☐	0
		T*м/град
UX1	☐	0
UY1	☐	0
UZ1	☐	0

(plates)

Options for adjoining elements of different types

(spatial core and volumetric elements)

- a) hinged, since many elements for solving the spatial problem of the theory of elasticity do not take into account the angles of rotation;
- b) pinching, when the rod is embedded at the appropriate length into the body;
- c) the use of a rigid body depicted in the form of a spider

About coordinate systems

When solving the ITU problem, various coordinate systems are used both to describe the calculation scheme itself and to provide the results of calculations, and to simplify the calculations themselves. Coordinate system of the calculation scheme When describing a problem, it is necessary to specify its coordinate system. It can be:

- three-dimensional problem of the theory of elasticity in space XYZ;
- a plane problem of the theory of elasticity in an arbitrary plane;
- bending plate in the plane XOY;
- a combined system in three-dimensional space XYZ, including rods, plates, shells and spatial bodies;
- the plane on which the inclined shell in question is plotted on the map.

Obviously, there are other options for specifying coordinate systems of the design scheme that are not listed above.

Stiffness Characteristics Coordinate System

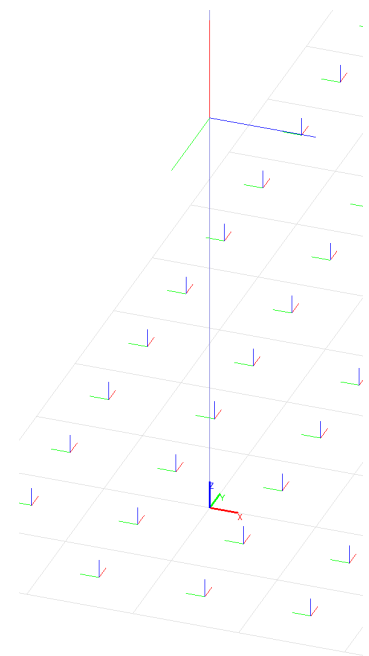
Already for orthotropic material, it is more convenient to set the directions of the main axes than to recalculate the stiffnesses, for example, in the coordinate system of the design scheme, while obtaining anisotropy. Even for a flat problem, in the most general case, it is easier to specify nine numbers than twenty-one.

Node coordinate systems

As a rule, displacement is the degree of freedom of nodes in the workflow. It is often necessary to identify oblique relationships. If you do not additionally introduce a node coordinate system, then for its implementation it is necessary to use special finite elements.

Coordinate systems from finite elements

When constructing a finite element, a local system is often introduced, which is defined only on it and selected for the convenience of appropriate constructions and calculations. Obviously, initially, in this coordinate system, all degrees of freedom of the element are considered and the corresponding calculations (constructions) in it are performed.



About coordinate systems

Coordinate system for setting loads

Installation of loads in modern computer systems is possible:

- in the global coordinate system – in the coordinate system of the main scheme;
- in special coordinate systems: cylindrical, spherical, etc.
- in the local coordinate system of the element: normal load on the surface of the shell, etc.

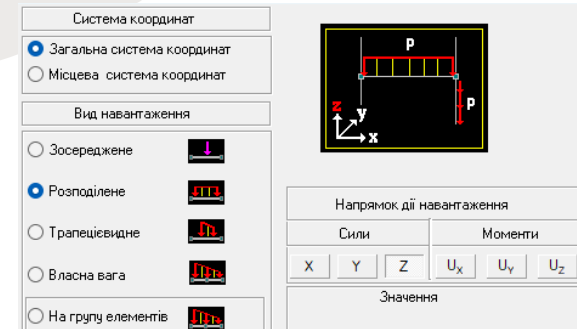
Coordinate system for the issuance of forces

During calculations, it is often necessary to obtain forces and stresses in the elements of the design scheme in special coordinate systems, which are due to design features: for example, the location of reinforcement.

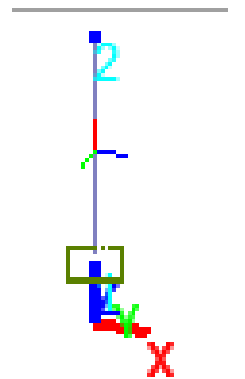
Features of transformations

All of the above coordinate systems must be taken into account when constructing finite elements:

- conversion of stiffness characteristics into a coordinate system for building a matrix of stiffness of an element;
- conversion of the stiffness matrix into a system (system) of coordinates of the nodes of the design scheme;
- conversion of loads defined in the general coordinate system into the element coordinate system.
- reduction of local loads on the element to nodal in their coordinate systems;
- conversion of calculated values of degrees of freedom of nodes into coordinate systems of elements;
- conversion of the design characteristics of the stress-strain state into specified coordinate systems.



N	M _k	M _y	Q _z	M _z	Q _y
T	T*м	T*м	T	T*м	T
-18,38	9,56e-006	-1,34	0,87	-0,27	-0,22



Three-dimensional problem of the theory of elasticity

To solve the three-dimensional problem of the theory of elasticity, finite elements of the geometric shape of a tetrahedron, a parallelepiped, a triangular prism are used. If the area of an element can be conformally displayed on them, then we are talking about isoparametric elements. The simplest elements have nodes that coincide with the vertices of polyhedra. To the features with increased approximation, nodes that lie on their edges are added.

Consider a stress-strain state at an arbitrary point of a three-dimensional body $x = \{x, y, z\}^T$, which is affected by external volumetric loads $f(x) = \{f_x, f_y, f_z\}^T$, under the influence of which movements occur $u(x) = \{u, v, w\}^T$.

Let's select an elementary parallelepiped around this point.

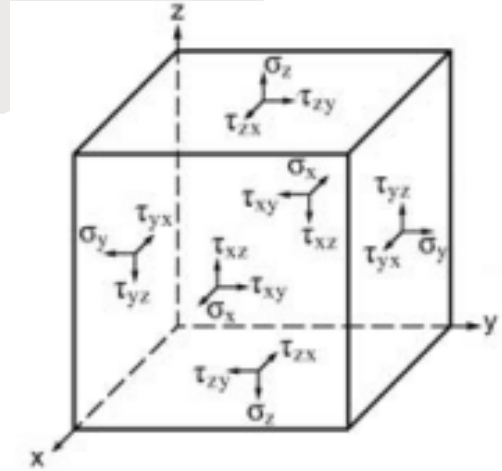
Stresses are applied to the faces of the parallelepiped, statically equivalent to the actions of the discarded parts of the body:

$\sigma_x, \sigma_y, \sigma_z$ — normal stresses parallel to the axes X, Y, Z ;

$\tau_{xy}, \tau_{yx}, \tau_{yz}, \tau_{zy}$ — shear stress in the corresponding planes.

If the volume of the parallelepiped tends to zero, then the stress state at point x is characterized by a vector of normal and shear stresses:

$$\sigma(x) = \{ \sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz} \}^T$$

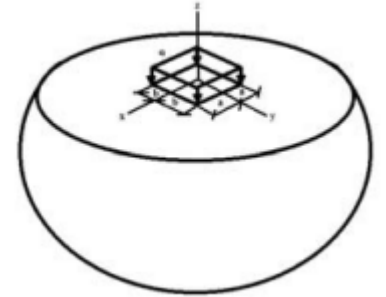


Stresses in a three-dimensional body

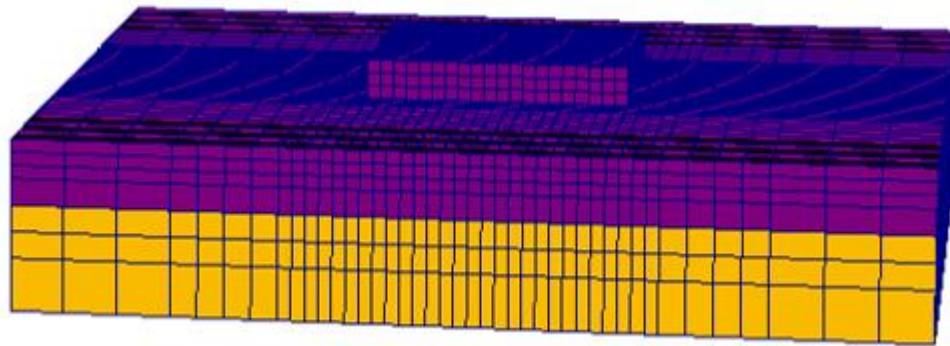
At the considered point, the equations of the Navier equilibrium have the form:

$$\begin{cases} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + f_x = 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + f_y = 0 \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + f_z = 0 \end{cases}$$

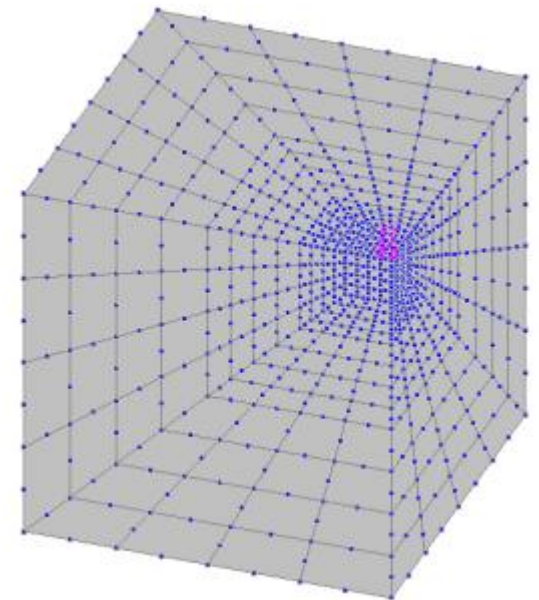
Modeling an elastic half-space in a three-dimensional problem statement



A feature of the calculation schemes for this problem is that it is impossible to describe an infinite area with elements of finite dimensions. Therefore, having determined the size of the calculated area, we thereby obtain a certain calculation error regarding the analytical solution.



Model of the "foundation-foundation-structure" system



Lev's problem in a finite-element axisymmetric model

Advantages and disadvantages of using volumetric elements for modeling bases

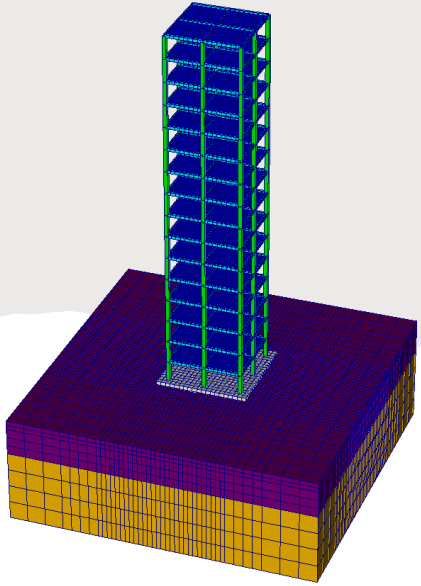
◆ **Model advantages**

- ✓ As elastic characteristics, the data of geological surveys are directly used - the modulus of deformations and the Poisson ratio;
- ✓ To assign geometric parameters of the base by the depth of the compressible thickness, known regularities are also used for a linearly deformable layer or for a linearly deformable halfspace with a limitation of the depth of the compressible layer;
- ✓ The calculations carried out indicate the coincidence of the results obtained on the displacements of the nodes of the day surface of the design model of the foundation with the average values of subsidence, calculated analytically both for the model of the linearly deformable layer and for the half-space within the 10%.
- ✓ Models on three-dimensional elements have a very high level of clarity both in the analysis of the deformed state and in the analysis of stresses in the substrate.

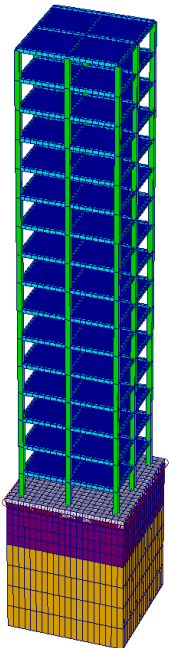
◆ **Disadvantages**

- ✗ The main disadvantage is the overestimation of the dissolving capacity of the foundation (further research is required to obtain adequate results of stresses in the foundation).
- ✗ It requires the use of volumetric elements that increase the labor intensity and dimensionality of the task.

Analysis of the modeling method using volumetric CE



1. The model overestimates the distributivity of the base outside the slab.
2. Under the edges of the foundation slab, fictitious shear forces occur, which lead to a general bending of the slab downwards, so the lower reinforcement of the slab is overstated.
3. Unlike the Winkler model, the model allows you to determine, in addition to contact stresses, the stress-strain state of the soil of the entire base.



1. The model does not take into account the dissolving capacity of the substrate outside the slab, which is unacceptable for cohesive soils.
2. The nature of the slab deformation and the stresses N_z under the slab are consistent with the Winkler model, but the subsidence of the base is overestimated to 50%.
3. The upper reinforcement of the slab, obtained according to this model, is overestimated due to the greater subsidence of the edges of the slab and the general bending of the slab upwards.
4. The stress-strain state of the soil is not plausible.

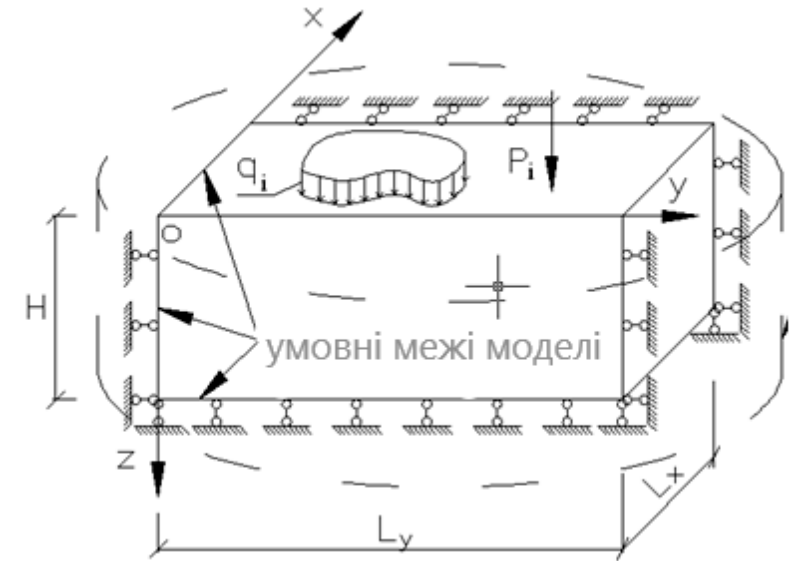
Substantiation of boundary conditions and deformation characteristics of the soil base model to obtain adequate results of the calculation of the "foundation-foundation-structure" system is one of the important scientific tasks of geotechnics.

Models of a linear-deformed layer of finite width and resolution

In the development of previous models and the elimination of their shortcomings, prof. I.Y. Luchkovsky and prof. Samorodov O.V. (2000) using the model of a continuous linear-deformed layer of finite resolution.

In this model, in addition to limiting vertical deformations at a certain depth H , it is also provided for limiting horizontal deformations at a certain distance from the place of application of the load in the plan ($L_x \times L_y$).

This model was proposed in order to eliminate the main drawback of the described linear models - the inability to solve two main calculation problems: to achieve compliance of the calculated subsidence and forces in large-sized slab foundations with the actual variation only by the modulus of deformation E_0 and the depth of the compressible thickness H_0 . The use of nonlinear models (especially for large-sized foundations) with more than two or three parameters only leads to the difficulty of conducting special soil tests with the interpretation of the results and even greater uncertainty and complexity of modeling.



Scientists have proposed a method for determining the main design parameters of a continuous linear-deformed layer of finite resolution (calculated layer thickness H_0 and design deformation modulus E_0) for modeling soil foundations of large-sized foundations:

- the actual (real) depth of the compressible thickness of H_ϕ is determined at an average pressure p along the foundation sole;

- The average subsidence of the SSP of the foundation slab is calculated, taking into account the determined actual depth of the compressible thickness H_ϕ ;

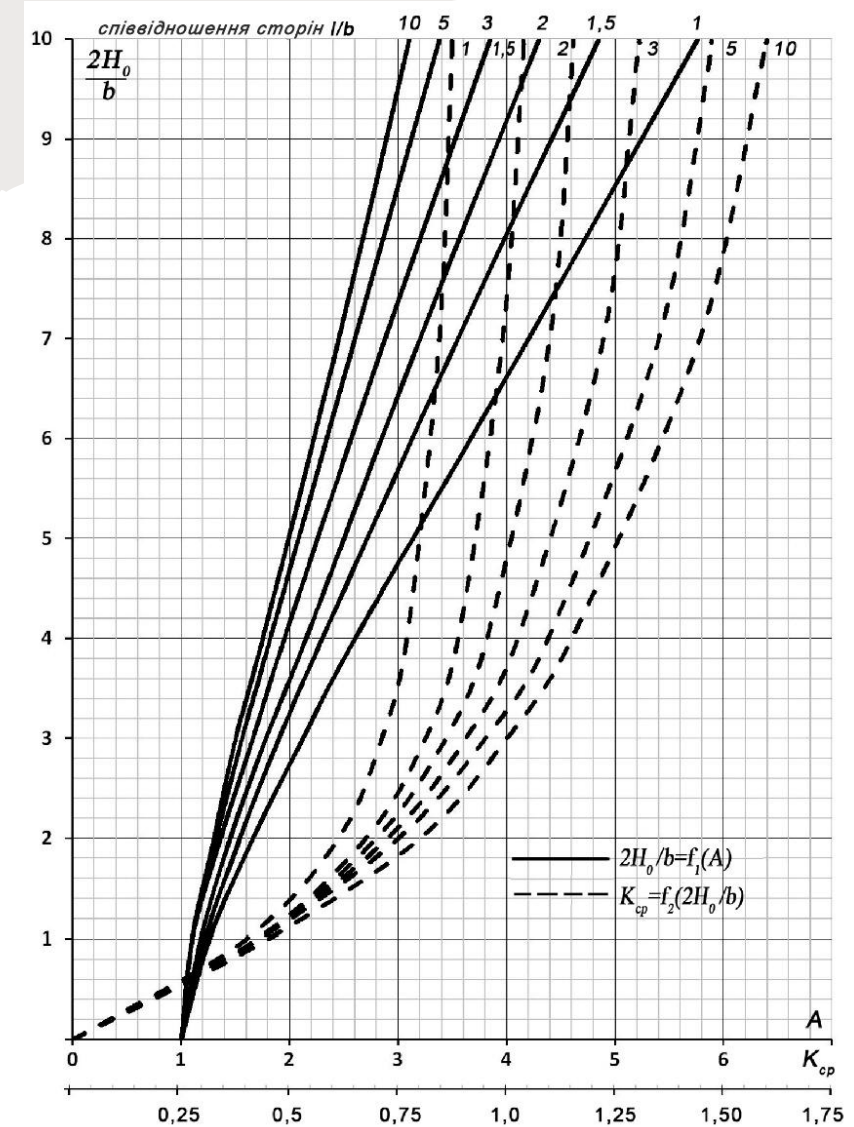
- the value of A is determined by the formula:

$$A = \frac{pH_\phi(1-\nu^2)}{s_{cp}E_\phi}$$

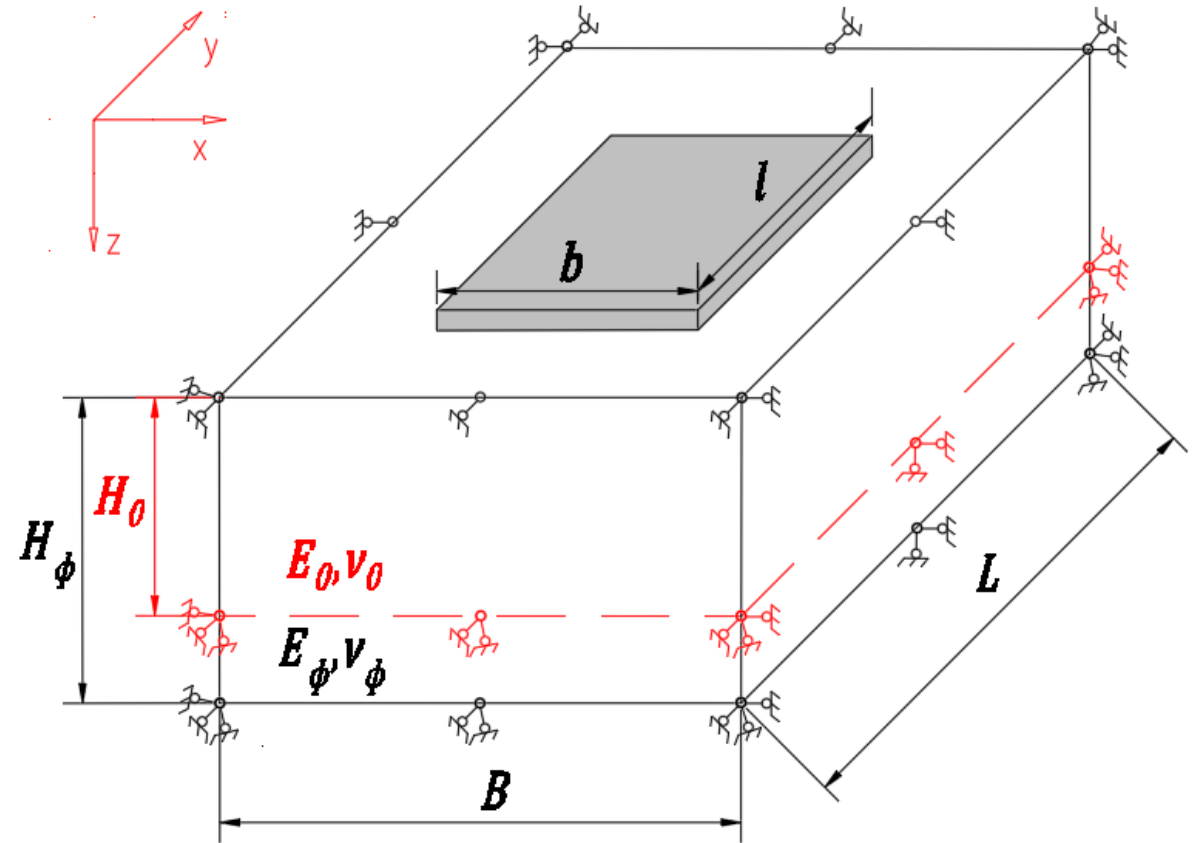
- by the found value A and the ratio of the sides of the foundation slab l/b according to the graph in Fig. 2 is the ratio $2H_0/b$ and K_{cp} ;

- the values of the main parameters of the model are calculated:

$$\left. \begin{aligned} H_0 &= \left(\frac{2H_0}{b} \right) \frac{b}{2}; \\ E_0 &= \frac{pb(1-\nu^2)}{s_{cp}} K_{cp}. \end{aligned} \right\}$$



The design thickness of the linear-deformed layer H_0 is less than the actual value of the compressible thickness H_ϕ and the design modulus of deformation of the linear-deformed layer E_0 is also less than the actual (given) modulus of soil deformation E_ϕ . Therefore, the model of the soil base with the design parameters (H_0 and E_0), in contrast to the actual parameters (H_ϕ and E_ϕ), required additional studies of the effect of reducing the layer thickness on the distribution of bending torque forces in the slab under other equal conditions.



This model was further developed in 2023 with the participation of prof. Samorodova O.V., Assoc. Prof. Aleksandrovykh V.A. and postgraduate student of the Department of State Pedagogical Service Gavryliuk O.V.

Based on the fact that at that time and to this day there is no unified approach to the dissolving capacity of soil foundations, therefore, the study of the influence of boundary conditions in the plan (according to the distribution angle α) of the model of a continuous linear-deformed layer of finite resolution (width for a flat problem) on the stress-strain state of foundation structures is an urgent task for adequate modeling of the "OFS" system.

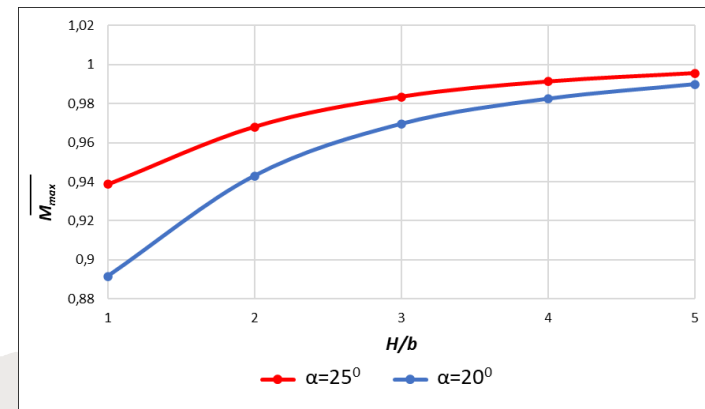
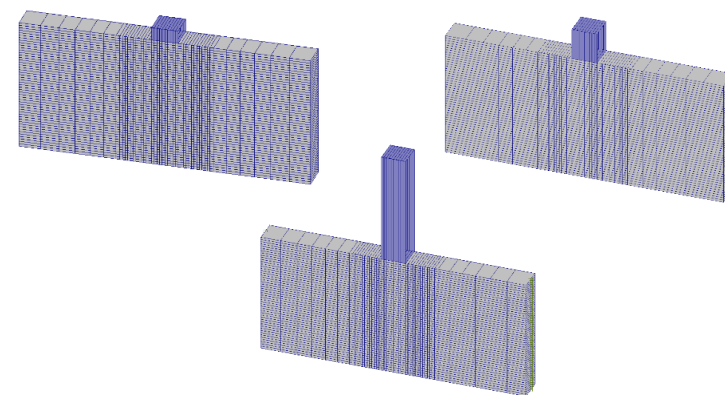
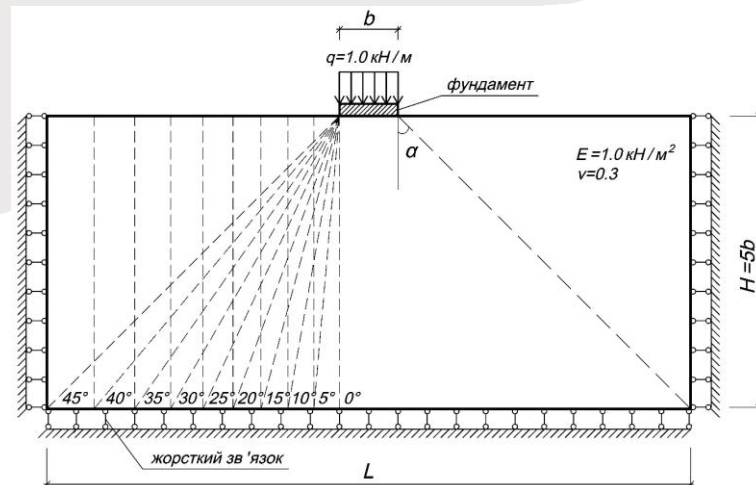
Initially, a flat formulation of the problem (plane deformation) was chosen to study the influence of boundary constraints in the plan of the model of a continuous linear-deformed layer of finite width on its resolution and deformability from the loaded foundation on the surface.

In the SCAD complex, numerical calculations of the stress-strain state of a uniformly loaded foundation of different relative bending stiffness (flexibility, t) interacting with a layer with a thickness H and a width of L equal to

$$L = b + 2 \cdot H \cdot \operatorname{tg}(\alpha_i)$$

with the restriction of linear displacements in perpendicular directions to the contour of the model.

Based on the analysis of the results of numerical calculations for a flat problem (plane deformation), the minimum allowable width of the model of a linear-deformed layer of finite width is substantiated, which takes into account the distribution of compressive stresses along the depth at an angle of $\alpha \approx 20^\circ \div 25^\circ$ to the vertical from the edges of the loaded foundation. At the same time, the width of the model according to the distribution angle of $\alpha \approx 20^\circ \div 25^\circ$ practically does not affect the average subsidence and maximum moment forces of the foundation in comparison with the angle $\alpha = 45^\circ$.

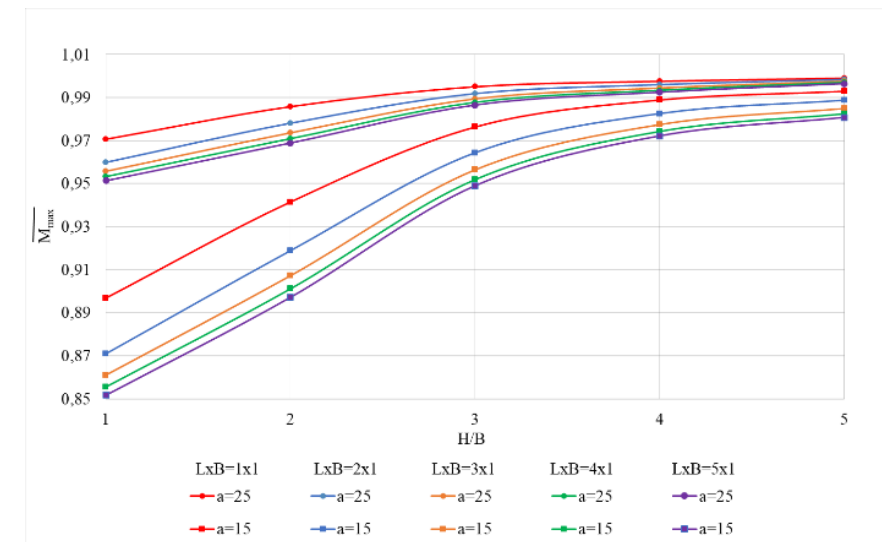
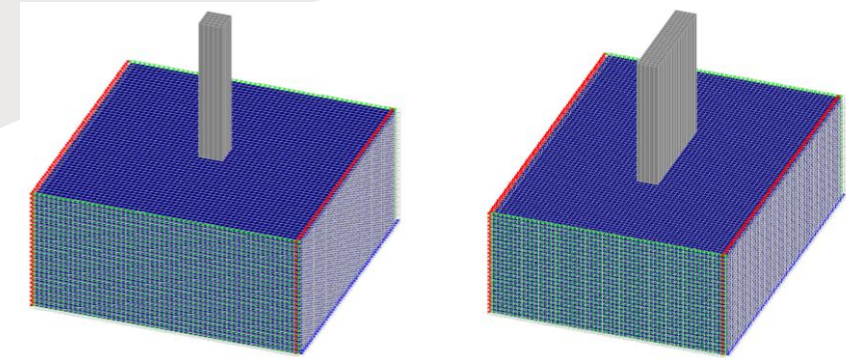


In continuation of the research performed by the authors for the flat problem, this study is performed in a three-dimensional setting.

The ratio of length and width ($L \times B$) in the plan of the slab foundation was variable and varied from 1×1 m to 5×1 m in increments of 1 m.

The model of the soil base was adopted as a linear-deformed layer of finite resolution symmetrical in terms of the model of the foundation slab. The thickness of the layer (H) was taken to be equal to $5B$ in incremental reduction to $1B$ (in increments of 1 m). The limitation of boundary conditions in the horizontal plane of the model occurred according to the angle of distribution α from 45° to 0° in increments of 10° .

Based on the analysis of the results of numerical calculations for the volumetric problem (volumetric deformation), the minimum allowable dimensions in the plan of the model of the linear-deformed layer of finite resolution are substantiated, which takes into account the distribution of compressive stresses at depth at an angle of $\alpha = 25^\circ$ to the vertical from the edges of the loaded foundation. At the same time, the overall dimensions of the model in the plan according to the distribution angle $\alpha = 25^\circ$ practically do not affect the average subsidence and maximum moment forces of the foundation in comparison with the angle of $\alpha = 45^\circ$. That is, the results of the studies of the spatial problem (finite resolution) demonstrated almost complete convergence with the results of the flat problem (finite width).

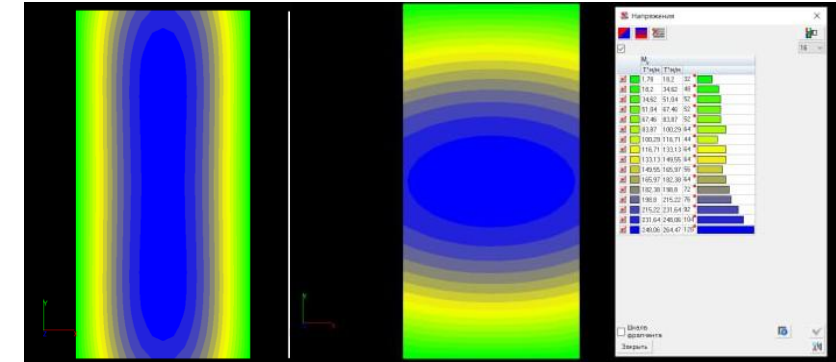
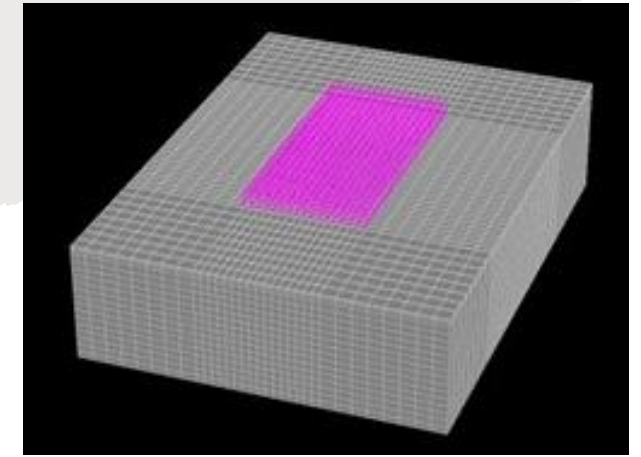


For numerical studies of the influence of the layer thickness of the model of a continuous linear-deformed layer of finite resolution on the distribution of moment forces in a uniformly loaded flexible slab foundation, the spatial formulation of the problem is chosen.

The purpose of the study is to numerically study the NDS of a uniformly loaded flexible rectangular foundation slab with a decrease in the layer thickness of the soil base model in the form of a continuous linear-deformed layer of finite resolution.

With the help of the SCAD complex, a finite-element model of the interaction of the soil base with the slab foundation was created.

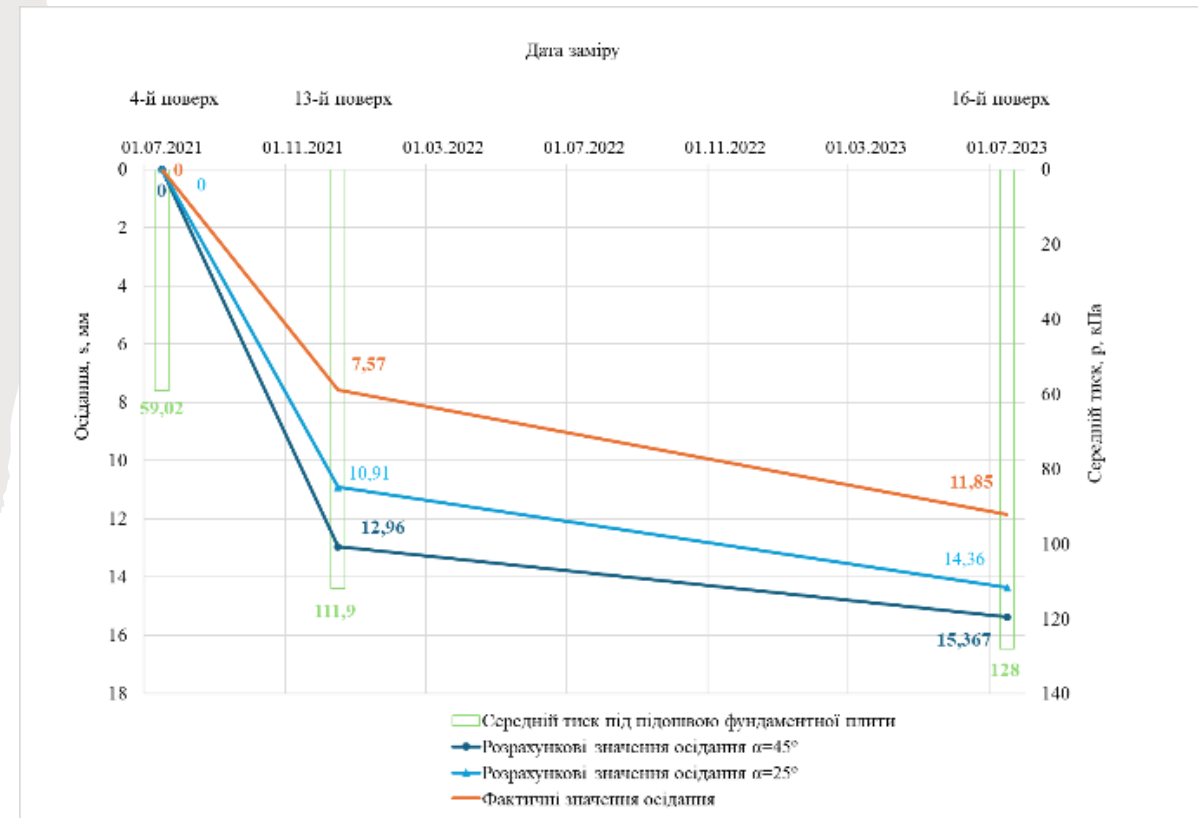
The analysis of the calculation results shows that with a decrease in the ratio of $H_0/N_f=0.25$ (the calculated layer thickness of the soil base model $H_0=4$ to the actual (real) compressible thickness of the soil base of the foundation $H_\phi=16$), according to the Luchkovsky-Samorodov method, the maximum moment forces along the orthogonal axes of rectangular foundations are reduced due to a decrease in the resolution of the soil base model and, accordingly, the reactions R under the slab at equal average subsidence s_p of the ssr. This can be of fundamental practical importance in the rational design of large-sized slab foundations, where reinforcement can be reduced by up to 50%.



This model was verified by comparing the results of calculations with full-scale observations of the subsidence of the building.

On the graph of the development of average subsidence, depending on the pressure under the foundation slab, it follows that the nature of the development of deformations determined by the calculation for both the distribution angle $\alpha=45^\circ$ and the distribution angle $\alpha=25^\circ$, is proportional to the nature of the development of subsidence determined during field observations. But they differ in absolute values. It should be noted that the distribution angle used $\alpha=25^\circ$ gives more adequate results, close to the actual subsidence.

A decrease in the angle of distribution of α according to the studies will lead to a change in the values of the average subsidence of the foundations and a decrease in the moment stresses in the foundation slab. Therefore, the distribution angle $\alpha=25^\circ$ can be used to model the soil mass of the base with ensuring the reliability of the values of moment stresses. This confirms the possibility of using the soil base model in the form of a linear-deformed layer of finite width to predict adequate deformations of the building on a large-sized slab foundation.



Advantages and Disadvantages of the Finite Resolution Linear Deformable Layer Model

◆ **Model advantages**

- ✓ obtaining the most adequate results of subsidence and efforts among linear models based on ITU;
- ✓ optimal balance between the dimensionality of the problem and the reliability of the calculation results;
- ✓ Verification of the model by full-scale observations of building deformation.

◆ **Disadvantages**

- ✗ requires further experimental verification of the influence of the selected calculated compressible thickness on the adequacy of the results of moment stresses in flexible plates.

Afterword

"There are few significant factors, but there are many trivial factors" ("The 20/80 Principle")

These statements lay the groundwork for refining the models: material factors (20%) should be evaluated as accurately as possible, and non-essential factors (80%) should be evaluated with much less precision.

The Pareto-Giordano principle