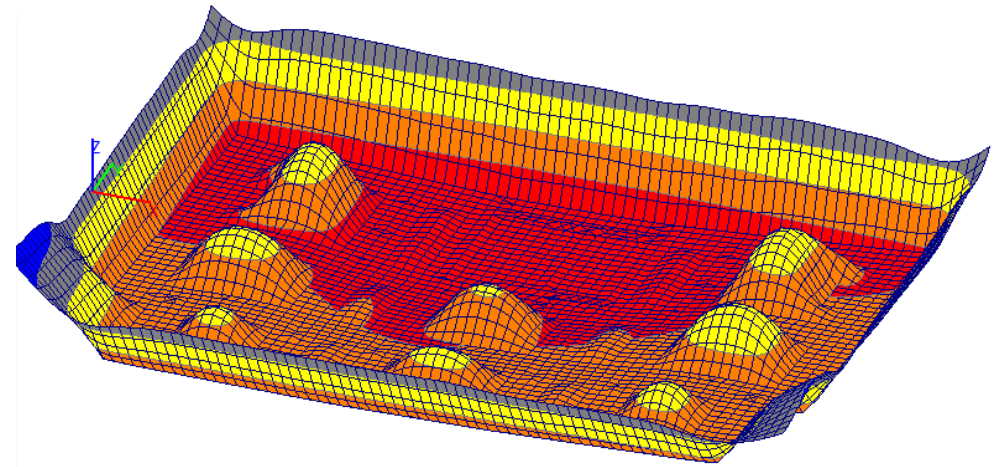


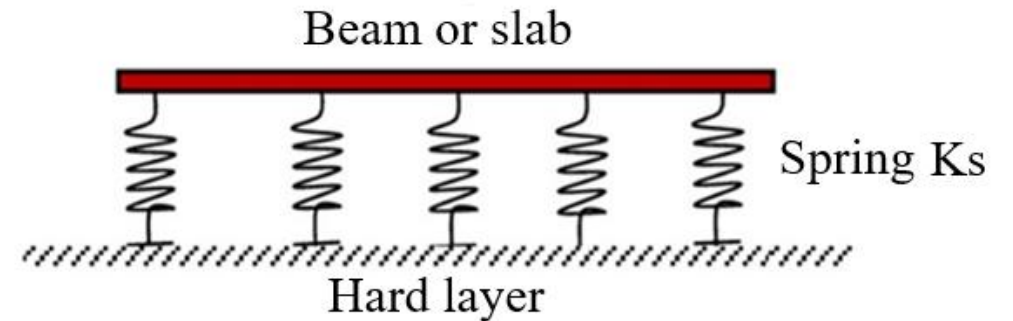
Topic 1.

Construction of flat models of the soil base



History of the development of soil models

Traditionally, since ancient times, a fixed non-deformable element is introduced into the calculation scheme, which is conventionally called "earth", with which the reference system is usually associated and for which the conditions of equilibrium are not required. And for many years, all the conditions of support were formulated as a rigid "anchorage to the ground". This tradition was first broken by Emil Winkler, who in his lectures on elasticity and strength considered the problem of a beam on an elastic base [Winkler, 1867]. This problem arose in connection with the analysis of the work of the railway rail track, and Winkler proceeded from the fact that each sleeper settles under load by a value determined by the rigidity of the base C , regardless of the behavior of the adjacent unloaded sleeper.



Winkler's model

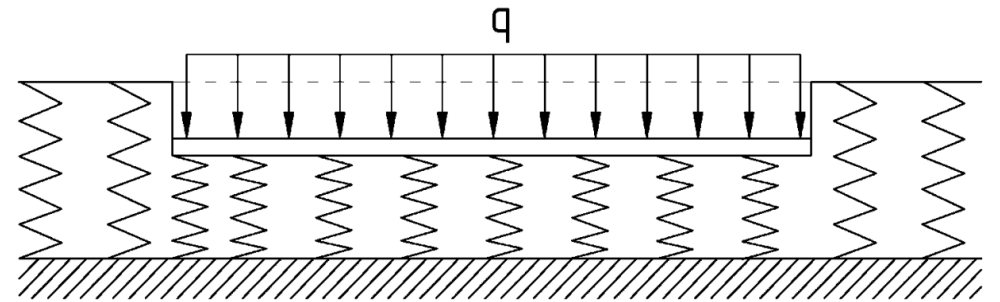
With the entry into a more active phase of technical progress and, as a result, an increase in the number of flexible structures on an elastic basis - first railway rails and sleepers, and then reinforced concrete foundations of buildings - there was a need for a more careful approach to assessing the reactive properties of soils. Naturally, there was an assumption that there is a direct relationship between subsidence and the reactive pressure of the soil.

Accordingly, the expression:

$$p = K \cdot Y$$

where Y is subsidence; K is the proportionality coefficient ("bed coefficient"), which depends only on the physical properties of the soil; p is the reaction pressure. Based on this position, Winkler's hypothesis appeared, which was first used to calculate railway tracks in 1868.

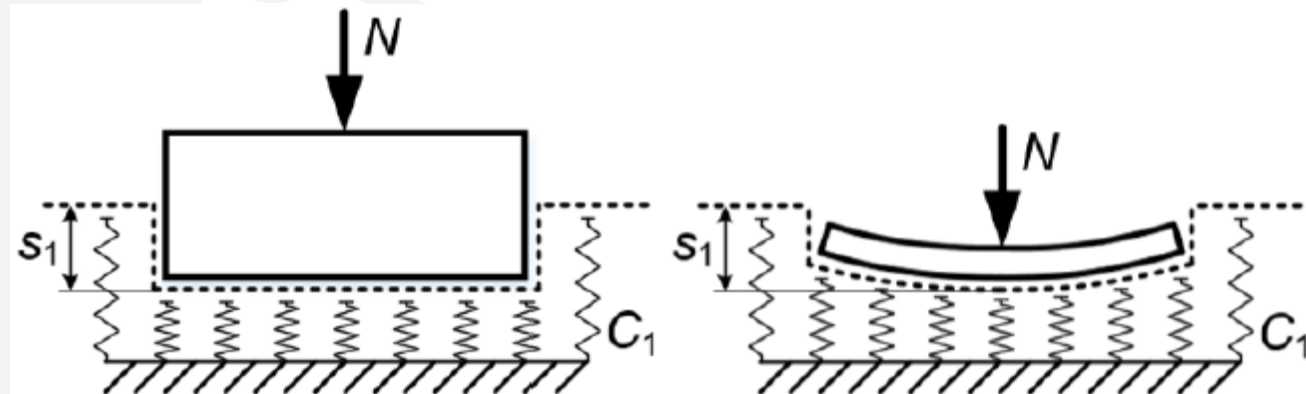
The coefficient K is determined experimentally and has the dimension of the force attributed to from the volume. For real soil conditions, the values of the bed K coefficient are determined very conditionally, therefore, in reference tables for the same soils, a range of possible changes in the bed coefficient is usually given. From a physical point of view, the Winkler model can be represented by a set of unrelated identical elastic springs resting on an absolutely rigid base.



For foundation subsidence, the Winkler expression can be written as follows: where S_z is the subsidence of the substrate, q is the distributed load on the strain that creates pressure under the foundation slab, and C_1 is the proportionality factor, most commonly referred to as the bedding ratio, which refers to how much load should be applied to produce a unit strain.

$$s_z = \frac{q}{C_1}$$

The behavior of the substrate approaches the Winkler model on very loose soils that poorly perceive shear forces in the soil or at high marginal stresses, causing a cut of the underlying soil layer due to the development of plasticity. The sedimentary funnel is not formed in this model.

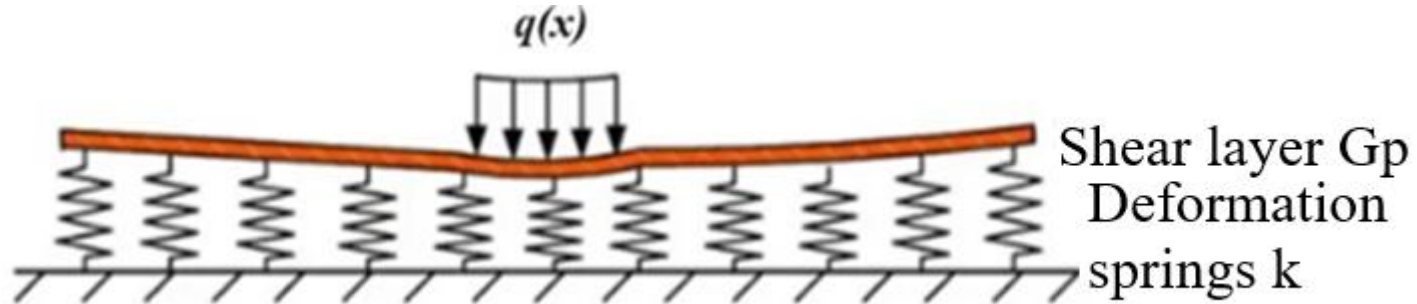


Disadvantages of the Winkler model:

- Deformation of the base occurs only in the zone of load applied to it. This reflects the real properties of loose and non-cohesive substrates quite well, but is completely unsuitable for dense and, especially, rocky foundations due to the lack of taking into account the deformations of the substrate that occur outside the zone of application of the load. According to observations, the soil settles and, therefore, is subjected to stresses outside the foundation;
- In practice, evenly loaded beams and slabs settle unevenly (as in the Winkler model), and, as a rule, are convex downwards;
- The value of the bed K ratio depends on the dimensions of the die used for the test to determine this value. The coefficient of bed K is larger, the smaller the area of the strain. Tabular values do not have defined values, but are represented by ranges of values.

The double contact model is constant

(Pasternak's model with two "bed coefficients")



It became a development of the Winkler model, which turned out to be too simplistic, since it did not take into account the interaction between adjacent points of the soil base.

The Pasternak model takes into account two parameters (stiffness coefficients):

The two-constant model takes into account two parameters (stiffness coefficients):

- **k** — bed coefficient (vertical elastic stiffness, as in the Winkler model, but not equal to it);
- **G** — shear interaction coefficient (characterizes the shear stiffness of the soil layer between elastic elements).

Thus, the soil is modeled not just as a system of independent springs, but as a layer of elastic material connected in a plane by shear bonds.

The equation of the relationship between the external pressure p and the deflection of the surface of the base w is of the form:

$$p = kw - G\nabla^2 w$$

де:

- p — Reactive soil pressure,
- w — Deflection of the base,
- k — Bedding coefficient (H/M^3),
- G — Offset or interaction parameter (H/M),
- $\nabla^2 w$ — Laplacian deflection (takes into account spatial changes in deformations).

- First member kw describes the vertical reaction of the soil (similar to Winkler).
- Second member $-G\nabla^2 w$ takes into account the shear interaction between adjacent sections of the base, i.e. the transfer of part of the load in the horizontal direction.
- This allows the model to reproduce smoother subsidence plots and realistic pressure distributions under slabs or beams.

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$$

For foundation subsidence, the expression of the two-contact model can be written as follows:

$$s_z = \frac{q}{C_1 - C_2 \cdot \Delta}$$

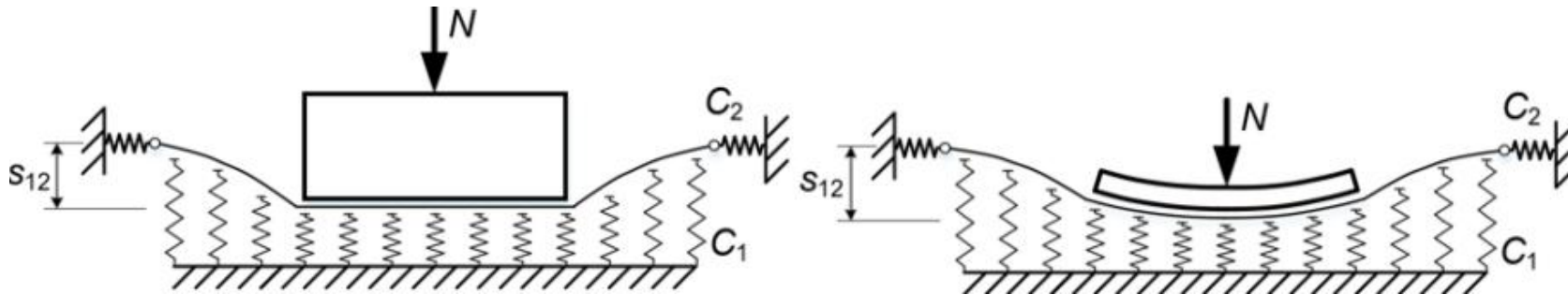
де

C_1 – Compression proportionality ratio, T/M³;

C_2 – Shear proportionality ratio

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad - \text{cinematographer Laplace } 1/\text{M}^2$$

The first parameter is analogous to the bed coefficient according to the Winkler hypothesis, but has a different numerical value - the more different, the more elastic the base with a flatter sedimentary funnel is. The second is that the shear coefficient takes into account the shear (shear) action of the elastic base. If the second parameter is taken as zero, then the model turns into a Winkler base.



Advantages and disadvantages of the two-constant contact model

◆ Advantages

- ✓ takes into account the interaction between the base points (unlike Winkler);
- ✓ allows you to describe realistic fields of deformation and pressures;
- ✓ has analytical solutions for simple geometries (beam, slab);
- ✓ can be generalized to cases of anisotropic or nonlinear bases.

◆ Restriction

- ✗ parameters k (C_1) and G (C_2) do not have clear physical analogues in real laboratory tests;
- ✗ the model remains linear — it does not take into account the plasticity and compaction of the soil;
- ✗ does not describe the change in the modulus of strain with depth.

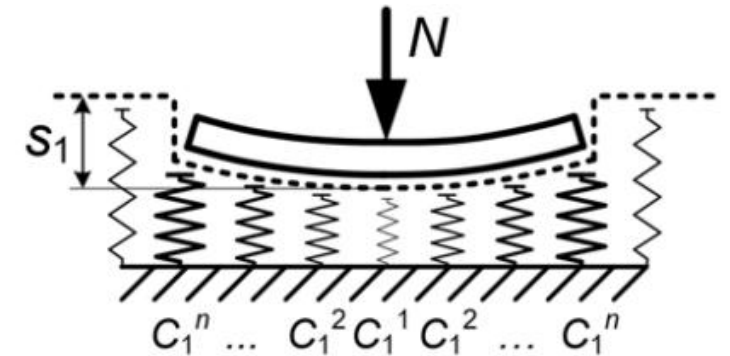
Model of variable bedding coefficients

The model with variable bedding coefficients is a development of the Pasternak and Winkler model, aimed at more accurately reproducing the heterogeneous properties of the soil base.

Its essence lies in the fact that the coefficients of elasticity (stiffness) vary in depth, in plan or in the direction of load, reflecting the real heterogeneity of the soil mass.

The need to take into account the variability of soil properties arose due to:

- Complex stratification of soils (alternation of layers of different densities and humidity);
- Effect of sealing under load;
- Dependence of deformation characteristics on stress state and depth.



The first research in this direction was carried out in the 1960s and 1970s. scientists - Klepikov S. M., Kerr A. D., Burmister D. M., Gibson R. E.

The equation of the relationship between the external pressure p and the deflection of the base surface w has the form similar to the two-parameter model, taking into account the change in the coefficients of the bed at different points of the slab:

$$p(x, y) = k_i w(x, y) - Gi \nabla^2 w(x, y)$$

de:

- k_i —bed coefficient that varies along the coordinate x ;
- Gi —shear interaction coefficient, can also be a function of the coordinate;
- $w(x)$ —surface deflection;
- $p(x)$ —pressure from the structure.

Bedding coefficients can be variable according to the following features:

- *By depth (vertical variability):*

$k_i = k(z)$, which describes the increase in stiffness as the depth increases.

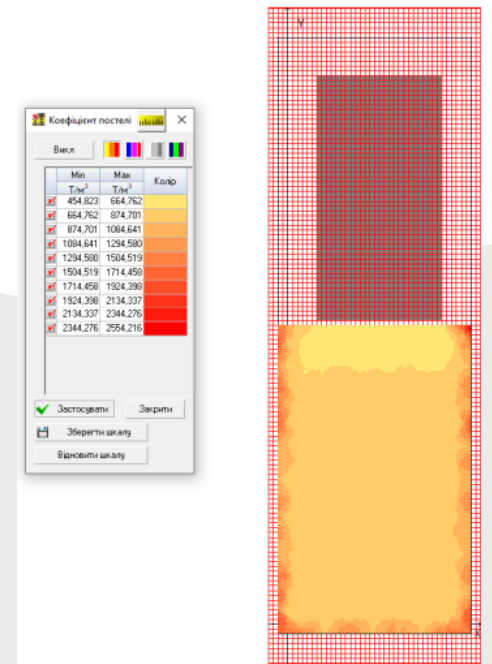
Typically used for dense soils or layers with compaction.

- *In the plane (horizontal variability):*

$k_i = k(x, y)$, In the plane (horizontal variability).

- *Nonlinear variability under load:*

$k_i = k(\sigma_z)$, when the modulus of strain depends on the stress level.

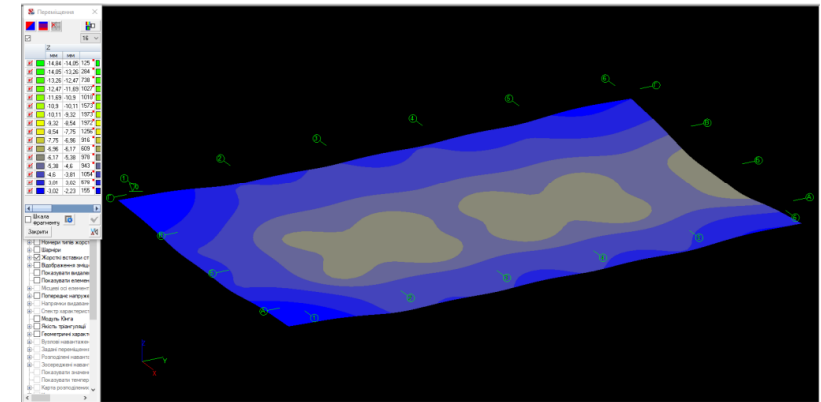


The model takes into account the fact that the soil does not react in the same way at all points, and its modulus of deformation increases with increasing depth and load.

Thus, it allows us to describe the realistic operation of foundations in a heterogeneous environment.

The subsidence of any point on the surface of the base (slab sole) can be written as:

$$s_z = \int_{z_f}^{z_d} \varepsilon_z dz \quad \varepsilon_z = \beta \frac{\sigma_z - \sigma_0}{E_e} \text{ при } \sigma_z \leq p_c; \quad \varepsilon_z = \beta \left[\frac{p_c - \sigma_0}{E_e} + \frac{\sigma_z - p_c}{E_0} \right] \text{ при } \sigma_z > p_c$$



де ε_z – Layer-by-layer soil deformations; z - vertical coordinate increasing downwards; z_f - Coordinate of the slab sole; $Z_d = z_f + H$ - Integration Lower Limit; H - Depth of compressed thickness.

Advantages and disadvantages of the model model with variable bedding coefficients

◆ Advantages

- ✓ increased accuracy in calculations in heterogeneous soil;
- ✓ the ability to simulate real geological conditions;
- ✓ suitable for numerical implementation (finite element method, finite differences).

◆ Restriction

- ✗ complexity of analytical solution;
- ✗ the need for detailed field or laboratory data;
- ✗ lack of uniform regulatory recommendations for the choice of function k_i .

Model of a linear-deformed half-space (half-plane)

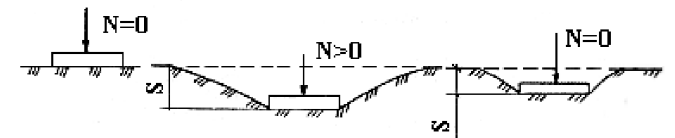
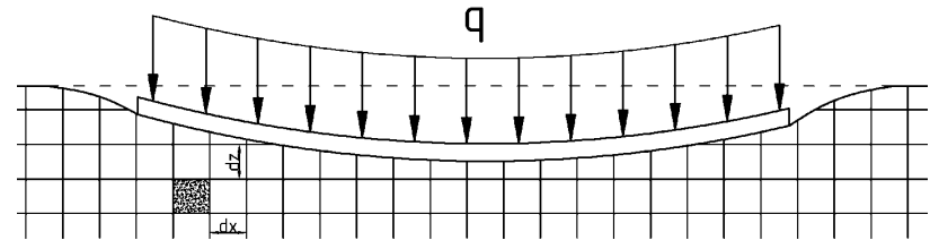
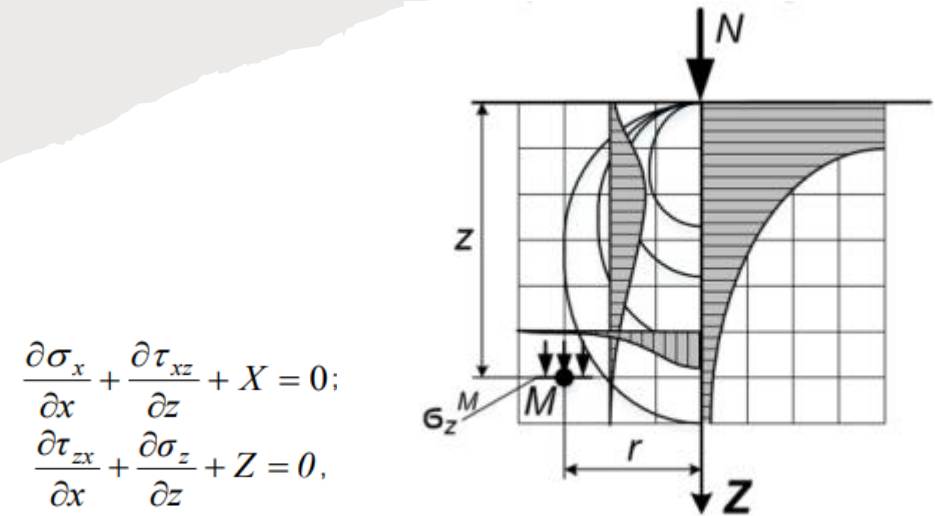
The model of linear-deformed half-space is a classical continual model of the soil base, based on the provisions of the theory of elasticity.

She describes soil as a homogeneous, isotropic, linear-elastic medium that occupies half space $z \geq 0$ i capable of deforming under the action of a load applied to its surface.

The first solutions to half space problems were:

- J. Boussinesque (1885–1887) — the problem of the distribution of stresses and displacements from a point vertical load on the surface of an elastic half-space;
- M. Flamant (1892) — Problem for surface concentrated horizontal load;
- further generalizations: Mindlin (1936) — for the load applied inside the volume, and the Burmeister (1943) — for a layered half-space.

Soils are characterized by the presence of predominantly plastic deformations. Therefore, the model of a linearly deformable half-space can be used only at the stage of one-time loading of the soil medium without further unloading, which is what happens in reality for most practical construction cases of static loads. When using the model of linearly deformable half-space, any problem is reduced to solving a system of equations, which, as is known from the course of the theory of elasticity, includes static equations, geometric relations, and physical equations.



Basic assumptions of the model:

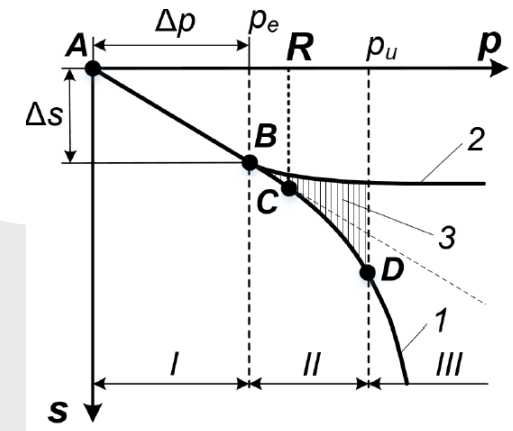
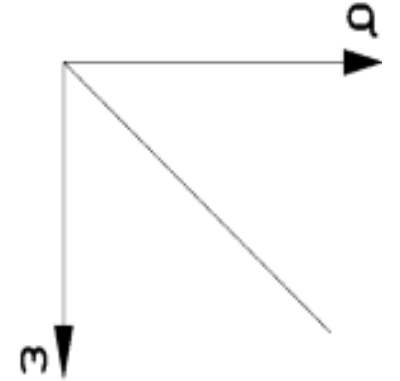
- Soil is a continuous medium obeying Hooke's law:
- The strains are small, and the relationship between stresses and strains is linear.
- The surface of the half-space is free from lateral stresses.

In contrast to discrete models (Winkler, Pasternak), half-space considers the soil as a continuous elastic medium in which deformations propagate spatially.

That is, the load at one point causes subsidence not only under itself, but also in neighboring areas, which corresponds to the real behavior of soils.

The model of linearly deformable half-space is plausible for soil substrates under flexible foundation slabs in the absence of stress concentrations at the ends of the cantilever sections of the slab, as well as at pressures under the slab well below the design resistance of the soil.

In this model of an elastic half-space, the diameter of the sedimentary funnel tends to infinity and it is not possible to take into account plastic shear deformations, due to which overestimated fictitious transverse stresses appear at the edges of the plate. A significant part of the deformation energy is redistributed from under the foundation to the formation of a sedimentary funnel in the adjacent soil mass.

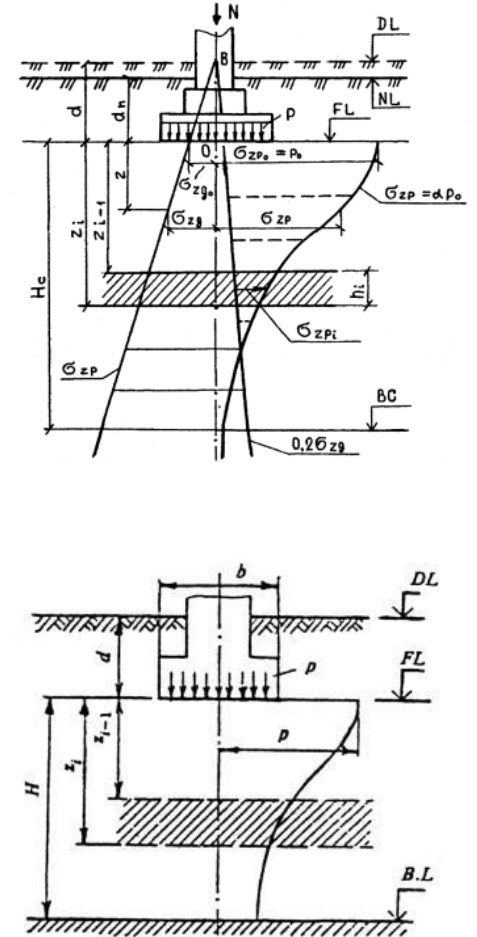


Linear-deformed layer model

- The model of a linear-deformed layer of finite thickness is a generalization of the model of a linear-deformed half-space, which takes into account the limited depth (thickness) of the elastic layer lying on a rigid or weak base.
- It is used to simulate the operation of the soil base when the depth of the compressed zone is comparable to the thickness of the layer or when the layer below has other mechanical properties (for example, clay on sand or vice versa).
- The limitation of the model of the linear-deformed half-space model to the physically justified depth of the compressible soil column in this model made it one of the main engineering models in calculating the subsidence of foundations and in taking into account the distributive properties of the foundation when solving contact problems. However, it should be noted that this model is designed only for determining vertical stresses and subsidence of the foundation loaded with flexible and rigid foundations. The distribution of horizontal stresses in the layer is not only not found, but also cannot be found, since the boundary conditions at the ends of the layer, as in the solutions for the model of a linear-deformed half-space, are not specified.

The model is used when:

- analytical determination of the stress-strain state in the soil under the foundations;
- calibration of bed coefficients for Winkler-Pasternak models;
- calculations of subsidence, contact stresses, shifts in geotechnical problems.



Advantages and disadvantages of the linear-deformable half-space and layer model

◆ **Model advantages**

- ✓ highest physical reliability among linear models;
- ✓ is based on a strict theory of elasticity;
- ✓ it is used to test numerical methods (analytical reference solutions);
- ✓ underlies the calculations of stresses and subsidence under foundations.

◆ **Disadvantages**

- ✗ mathematically complex — analytical solutions are possible only for simple cases (point or axisymmetric load);
- ✗ the occurrence of fictitious excessive reactions at the edges of foundation structures, and momentary efforts;
- ✗ not suitable for nonlinear deformations or plastic zones.

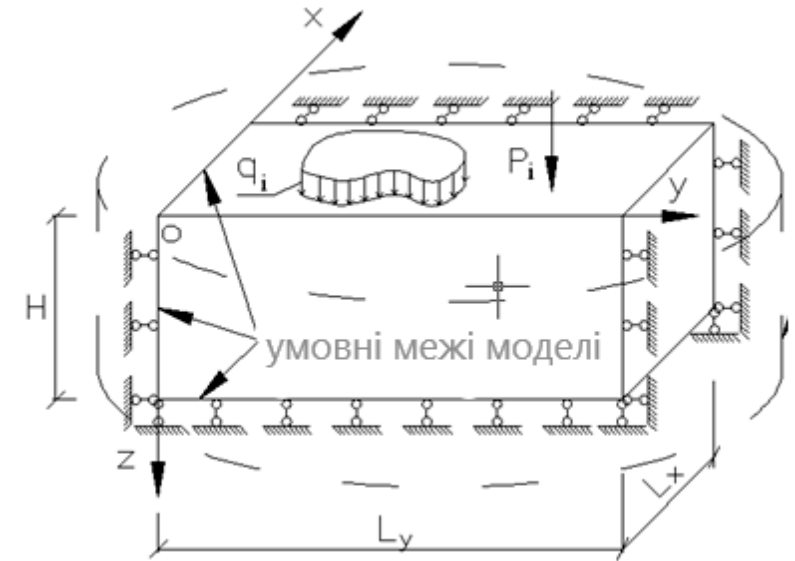
Model of a linear-deformed layer of finite width

In the development of previous models and the elimination of their shortcomings, prof. I.Y. Luchkovsky and prof. Samorodov O.V. (2000) using the model of a continuous linear-deformed layer of finite resolution.

In this model, in addition to limiting vertical deformations at a certain depth H , it is also provided for limiting horizontal deformations at a certain distance from the place of application of the load in the plan ($L_x \times L_y$).

This model was proposed in order to eliminate the main drawback of the described linear models - the inability to solve two main calculation problems: to achieve compliance of the calculated subsidence and forces in large-sized slab foundations with the actual variation only by the modulus of deformation E_0 and the depth of the compressible thickness H_0 . The use of nonlinear models (especially for large-sized foundations) with more than two or three parameters only leads to difficulties in conducting special soil tests with the interpretation of the results and even greater uncertainty and complexity of modeling.

In the flat formulation of the problem, this model is called the Model of a Linear-Deformed Layer of Finite Width.



This model was further developed in 2023 with the participation of prof. Samorodova O.V., Assoc. Prof. Aleksandrovykh V.A. and postgraduate student of the Department of State Pedagogical Service Gavryliuk O.V.

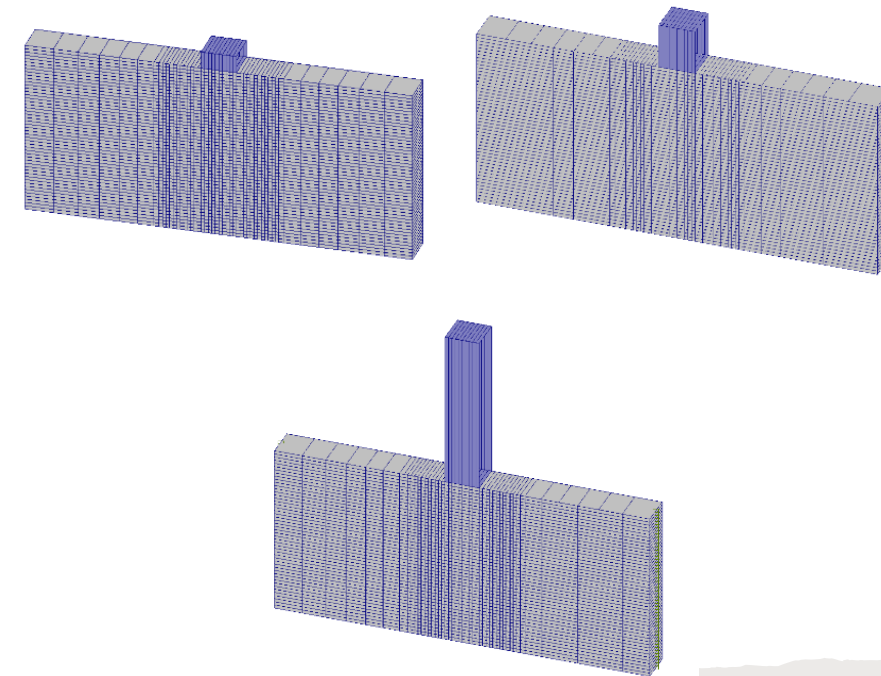
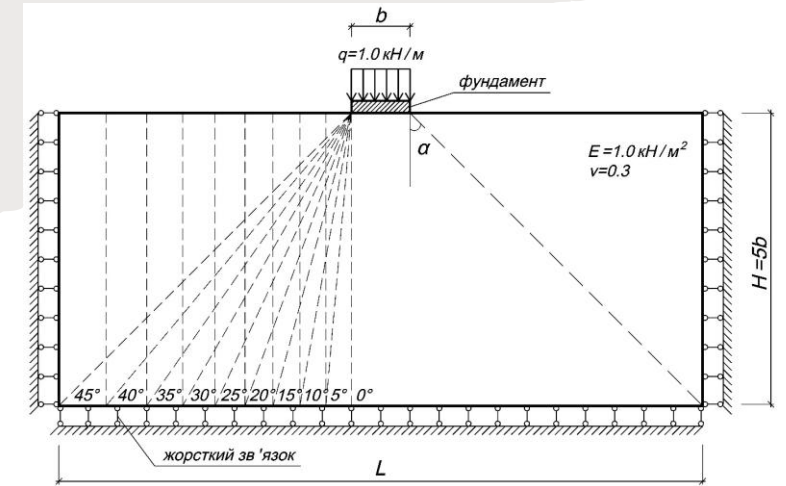
Based on the fact that at that time and to this day there is no unified approach to the dissolving capacity of soil foundations, therefore, the study of the influence of boundary conditions in the plan (according to the distribution angle α) of the model of a continuous linear-deformed layer of finite resolution (width for a flat problem) on the stress-strain state of foundation structures is an urgent task for adequate modeling of the "OFS" system.

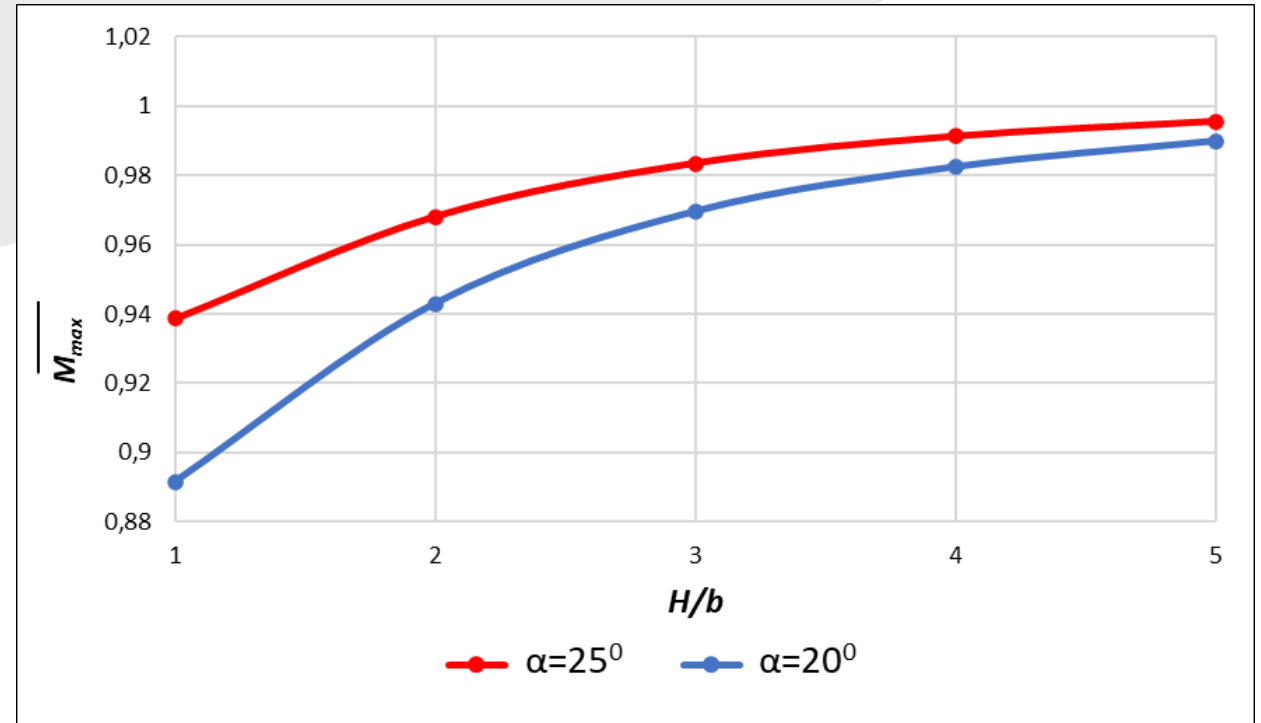
Initially, a flat formulation of the problem (plane deformation) was chosen to study the influence of boundary constraints in the plan of the model of a continuous linear-deformed layer of finite width on its resolution and deformability from the loaded foundation on the surface.

In the SCAD complex, numerical calculations of the stress-strain state of a uniformly loaded foundation of different relative bending stiffness (flexibility, t) interacting with a layer with a thickness H and a width of L equal to

$$L = b + 2 \cdot H \cdot \operatorname{tg}(\alpha_i)$$

with the restriction of linear displacements in perpendicular directions to the contour of the model.





Based on the analysis of the results of numerical calculations for a flat problem (plane deformation), the minimum allowable width of the model of a linear-deformed layer of finite width is substantiated, which takes into account the distribution of compressive stresses along the depth at an angle of $\alpha \approx 20 \div 25^\circ$ to the vertical from the edges of the loaded foundation. At the same time, the width of the model according to the distribution angle of $\alpha \approx 20 \div 25^\circ$ practically does not affect the average subsidence and maximum moment forces of the foundation in comparison with the angle $\alpha = 45^\circ$.

Advantages and Disadvantages of the Finite Resolution Linear Deformable Layer Model

◆ **Model advantages**

- ✓ obtaining adequate results of sedimentation and effort among flat linear models based on ITU;
- ✓ optimal balance between the dimensionality of the problem and the reliability of the calculation results;
- ✓ confirmed by the convergence of the results with further studies of volumetric models.

◆ **Disadvantages**

- ✗ limited by a flat problem statement.

Afterword

"All models are wrong, but some of them are useful"

Fallacy is not terrible if the model is plausible. A plausible model becomes useful if its parameters are calibrated against experimental data, obtaining a law, formula, or algorithm.

George E. P. Box